

**UNIVERSITI TEKNOLOGI MARA**

**QUANTUM PROPERTIES OF KERR  
NONLINEAR DIRECTIONAL  
COUPLERS VIA PHASE-SPACE  
REPRESENTATION**

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Thesis submitted in fulfillment  
of the requirements for the degree of  
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## **AUTHOR'S DECLARATION**

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any other degree or qualification.

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## ABSTRACT

Quantum statistical properties of interacting optical fields in Kerr nonlinear couplers may be regarded as one of the most fundamental problems yet to be solved in quantum optics. As an analytical solution to this system is not always possible, a semi-analytic or direct numerical solution must be employed in order to obtain a more complete description of their evolution. This study investigates the quantum properties of four-mode couplers composed of two and four Kerr nonlinear waveguides, both with and without cavity set up via phase space representation. In these systems, the electromagnetic fields are described by their Hamiltonians, while the time-evolution of the systems are described by the Von-Neumann equation. Following standard techniques, the master equations for the density matrices of the systems are obtained and further converted to the corresponding classical Fokker-Planck equations using both positive  $P$  and Wigner representations. The corresponding set of Langevin Stochastic equations is then obtained from the Fokker-Planck equations by means of Ito rules. Finally, the systems are integrated numerically and averaged over many trajectories to get the relevant information. The effects of self-action nonlinearity, cross-action coupling and the initial energy of the coherent light on the evolution of the field quadrature variances are investigated. Furthermore, the influences of multimode interaction on the dynamics of the squeezed states are discussed. In this research, we show that the system may produce squeezed states in both quadrature variances for all calculated parameters. The multimode two channel model can provide better squeezing in contrast to the conventional device. The amplitude oscillation of maximal squeezing may be amplified by manipulation of the initial state of the coherent light in the first mode, and the range may be extended through multimode interaction. When the model is contained in a cavity, the system is able to generate better amplitude oscillations and squeezing range with leaf-revival-collapse-like behaviour; with the whole oscillation able to evolve below the standard quantum limit. Regular photon transition and self trapping effects of the interacting modes in the four channels model are simulated, and squeezing is predicted for various initial intensities of the coherent field and nonlinear coupling parameter  $g$ . The cavity setup caused the system to undergo fluctuation amplification and the potential for different types of squeezing to be generated is observed. Depending on how we choose the interaction parameter values, the system may generate better squeezing in comparison to the cavity-less model.

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